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QUARTERLY PROGRESS REPORTS NO. 1, 2 and 3

MEASUREMENT OF DROP SIZE DISTRIBUTION

AND LIQUID WATER CONTENT OF NATURAL CLOUDS

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November 1, 1950 to February 28, 1951

TABLE OF CONTENTS

MEASUREMENT OF DROP SIZE DISTRIBUTION AND LIQUID WATER CONTENT

OF NATURAL CLOUDS*

I. Abstract

A search of the literature on methods of measuring drop size and total liquid water content in natural clouds shows no presently developed means which will continuously record both quantities from aircraft operating at moderate speeds and altitudes, say 200 mph and 20,000 ft.

Exploratory examinations and calculations have been made on two methods of measuring drop size. Both have been rejected temporarily. Computations on others are in progress. Experimental investigation of one of the methods now under study has begun.

II. Report

A. Statement of the Problem

The purpose of this research project is to discover and develop new or improved means for measuring the number and sizes of water droplets composing natural water clouds in the atmosphere, from aircraft in flight. An independent means of measuring the total liquid water content of clouds is also required.

Both measurements have been attempted in the past. Several instruments for each have been constructed but found wanting in various respects. A history of previous work may be found in the references

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* The contract dated June, 1950 was not finally accepted by both parties until November, 1950. This report is submitted to cover the actual contract period November 1, 1950 to February 28, 1951.

listed in Section A of the bibliography, Appendix III.

Measurements in fog (stratus cloud) at the earth's surface have shown (1) that droplet size can vary from 1 to 150 microns, (2) that most frequent sizes, on a volume median basis, are in the order of 20 to 90 microns, and (3) that total counts may reach as high as 6 per cubic centimeter (12)[*] in ground fog and 400 per cc. in clouds (4).

Direct measurement of total volume of liquid water and computations from drop size and count indicate that liquid water content may reach as high as 1.0 gram per cubic meter in clouds.

Airborne instruments moving at 100 meters per second (225 miles per hour or 20,000 feet per minute) and sweeping a cross-section area of 1 square centimeter would need to count up to 4×10^6 droplets per second and catch up to 0.01 gram of water per second in a cloud of 400 droplets per cc. and 1.0 gm/cu. mm.

The specification of 1 square centimeter as the swept area is arbitrary. Choice of any sample volume less than the whole cloud implies either an assumed knowledge of distribution of size and count in the cloud or measurement along a very large number of paths in the cloud. Since little is surely known about size and count distribution, an arbitrary choice must be made. In breaking the circle by setting a sample swept area and volume, great weight is necessarily given to the difficulties of counting and sizing such large numbers of drops as 4×10^6 per second. Future observations may show that a cross-section area of,

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[*]Numbers in parenthesis refer to references in appended bibliography.

say, 10 square millimeters will sample any region of a cloud with satis-
factory accuracy.

Many previous instruments, particularly of the impact and sedi-
mentation types (cylinders, slides, etc.) have had collecting surfaces
or nozzles so much larger than cloud droplets that their collection
efficiencies have been low and very variable with respect to drop size.
Corrections for the lost count have been based in large part on unverified
theory.

It is considered desirable therefore to set a tentative require-
ment that the new method should not contact or otherwise alter the size,
shape or relative position of the drops previous to or during their
counting and sizing. This requirement immediately sets strong limits on
the kind of measuring method possible, perhaps to a prohibitive degree.
Experience indicates, nevertheless, that it is a desirable requirement.

B. Methods of Attack

1. A common method of droplet size and counting is the accu-
mulation of all drops in a swept volume on a surface variously oriented
and exposed to the natural wind or carried by a vehicle.

    a. The surface may be normal to the air stream at wind
        or airplane speeds.

    b. The surface may be normal to an air current of reduced
        air speed as in an expansion chamber.

    c. The surface may be horizontal and parallel to the wind.

In the latter two forms, gravity or centrifugal force may distribute the drops according to mass (and size). To avoid subsequent evaporation of the catch, the surface must be maintained at air temperature and saturation humidity while it is photographed or the droplets must be imbedded into a surface coating so that they or their replicas are retained in full number.

The sedimentation or glass slide method has been very popular because of its apparent simplicity. Its weaknesses include (1) poor sampling efficiency due to the small exposure time with respect to the time elapsing while changing slides, (2) the time spent in tedious evaluation of count and (3) the variability of collection efficiency with air speed and drop size. Assuming 20 per cent increase in cross-section area on striking, a slide one centimeter square will become 50 per cent covered when exposed to 80 micron drops for 0.0005 seconds at an air speed of 225 miles per hour, in a typical cloud. The total count to be made is then 7000 drops.

2. A frequently suggested means of measuring droplets size in the free air is by observation of optical diffraction rings or coronae. This method yields only the mean drop size. It is obviously most effective with drops of uniform size and presents difficulties when used in natural clouds having a wide drop size range. Furthermore, coronae are strong and easily measured only for drops less than 20 microns in diameter.

More detailed and quantitative measurements of forward, side and back scattered light are possible with such optical detectors as

photo-cells and cameras. Elaborate measurements at various angles from the beam axis and in variable frequency monochromatic beams may lead directly to drop size distribution. Further investigation of these methods is described below.

3. Direct photography of cloud droplets in a cloud is possible, given sufficiently high light intensity, film emulsion speed, shutter speed, sharpness of focus and magnification. Current techniques of stroboscopic illumination may make this method feasible, even at aircraft speeds. This method will be studied in some detail.

4. Detection of individual drops in succession in a path of very small swept area may be obtained

a. by counting pulses of scattered light from each
   particle in transit,

b. by counting pulses of electric charge on a contacting
   probe in the air stream, and

c. by counting pulses of electric current induced in a
   magnetic coil or electrostatic condenser surrounding
   the air stream and the passing charged or uncharged drops.

Sizing of droplets by measuring the pulse magnitude in each method may also be possible.

The light pulse method (a) is difficult because of the very small amount of light energy scattered by a single drop. The contact pulse method (b) is weak because it does not account for the initial natural charge on the cloud droplet which may be any value between the positive and negative maxima determined by drop size and atmospheric

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MEASUREMENT OF DROP SIZE DISTRIBUTION AND LIQUID WATER
CONTENT OF NATURAL CLOUDS — AND APPENDIXES A-D — NOV 1,
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DROPS, LIQUID — MEASUREMENT   METEOROLOGY (30)
CLOUDS — WATER DROPLETS   AQUEOUS VAPOR AND
                                     HYDROMETEORS (6)

leakage resistance. Also the probe may show a variable collection efficiency for the smaller droplets.

The induced pulse method (c) is difficult because of the very small induced currents or charges caused by water droplets, also the dependence of these currents or charges on the location of the passing drop within the pick-up coil or condenser. In this method, the induced effects may also be functions of the charge on the drop, which is randomly variable as stated above. A pre-charge to maximum in an up-stream device would remove this variation.

A derivative of the induced pulse method using charged part-icles is one in which the change in permeability or dielectric constant of the air due to the presence of the water drop produces a measurable change in inductance or capacity. Since the permeability of air and water are in the ratio 1.0001 to 1.0000 or less, this method may be re-jected. The ratio of the dielectric constants of water and air is 81. A preliminary study of the probable sensitivity of a capacitively in-duced pulse method working on this principle has been made and is reported below.

C. Preliminary Investigations of Certain Proposed Methods

1. A study of the feasibility of an electrostatic charged cloud particle counter in which each droplet is first charged to its maximum and then detected and classified as to diameter by its charge.

The work of Ladenburg (19) and others indicates that the charging rate of spherical dielectric particles passing through a corona discharge is independent of the size of the particles and that the

maximum allowable charge on spherical particles varies directly as the square of the diameter.

The equation relating these variables is given by Ladenburg as the following:

$$Q = (1 + 2\frac{k-1}{k+2})\ \text{Er}^2\ \frac{\pi m_i d_i et}{(1 + \pi m_i d_i et)} \tag{1}$$

where Q = charge r = particle radius

    k = particle dielectric constant m_i = ion mobility

    E = corona field strength d_i = corona ion density

    e = electron charge t = time

Stated in another way, equation (1) predicts that each cloud droplet will pick up the same proportion of its maximum allowable charge during the same exposure to a corona discharge. Since k, e, π, m_i, and d_i, are known or measurable, equation (1) can be condensed to

$$Q = Ar^2\ (\frac{Bt}{1 + Bt}) \tag{2}$$

Droplets in the atmosphere usually carry an existing charge which can be associated with an initial charging time t_o. If the initial charge is zero, or any magnitude of the same sign as the final maximum charge, the equation may be rewritten as

$$Q_o + \triangle Q = Ar^2 \left(\frac{B(t_o + \triangle t)}{1 + B(t_o + \triangle t)}\right) \tag{3}$$

where $\triangle t$ is the time of exposure to the corona discharge and $\triangle Q$ is the increase in charge on the droplet of radius r. Solving for r^2 gives

$$r^2 = \frac{(Q_o + \triangle Q)}{A\left[\dfrac{B(t_o + \triangle t)}{1 + B(t_o + \triangle t)}\right]} \tag{4}$$

From this last form, it may be seen that there are two possible ways of determining r; first, by making $B(t_o + \triangle t)$ much greater than 1 and measuring Q_{max} only, and second, by measuring Q_o, $(Q_o + \triangle Q)$ and $\triangle t$.

The first method requires that $B \triangle t$ be made large. Suppose an accuracy of 98% is desired in radius r. All droplets must then be charged to at least 95% of maximum. Setting 95% equal to $(Q_o + \triangle Q)$ in equation (3) and using Fuch's (8) value of 90 for B, $\triangle t$ is found to be 0.21 seconds. If the droplets have a random distribution of charge, some will have a maximum charge of opposite sign to the charging corona. As a first approximation, assume that twice 0.21 or 0.42 seconds would be required to charge all drops to 95% of maximum.

Suppose the drop counter to be carried in an aircraft at 200 miles per hour (90 meters per second). The length of tube required to contain the corona discharge mechanism would be 37.5 meters, for 95% of full charge on all drops. Such a length is inconvenient if not impossible on conventional aircraft. A tube length reduction of 2/3 would require an increase of B of 3 or more. Such an increase would almost certainly carry d_i and E beyond the breakdown values, even at sea level. At high altitudes reduced pressure sets a further limit on the maximum value of corona potential and B.

It might be suggested that exposure time be increased by an expanding cross section in the corona discharge tube. While velocity could be satisfactorily reduced, adiabatic temperature effects would arise which might lead to condensation and growth of the droplets. The magnitude of these effects requires further study. Similar conditions exist in the housing of the reconnaissance aircraft psychrometer ML-313. (35).

The second method, involving measurement of Q, $(Q + \Delta Q)$ and Δt, allows partial charging in a short tube. Suppose the tube be 1 meter long and the air velocity still 90 meters per second. The exposure time Δt is then 0.01 seconds. Assume instrumental errors in charge measurement of 5%. It can be shown that the best theoretical accuracy in droplet radius will be at least 95%. The practical difficulties are many. Total charge must be measured twice on an individual drop. At a given time there are, say, 50,000 particles in the tube, at a cloud drop density of 500 per cc. The problem of droplet identification during the period Δt is great.

2. Counting and sizing of water droplets by variation of the dielectric constant in a condenser surrounding an air stream containing natural cloud.

The capacitance of a parallel plate electric condenser is given by the expression

$$C = \frac{KA}{3.6 \pi x} = 0.0885 \ KA/x \qquad (5)$$

where A and x are plate area and separation and K is the dielectric constant, which varies with the substance separating the plates.

By passing a stream of air containing water droplets between the plates, the capacitance can be varied in proportion to the masses of air, water and water vapor in the stream. If the condenser is large, the gross liquid and gaseous water content of the air may be measured. If the condenser is small, it may be possible to observe the passage of single droplets of water between the plates, with consequent determination of drop and distribution in the air sample. It remains to determine if the effect is measurable with present or probable future instrumentation.

Suppose that the air has a particle density of 1000 droplets per cubic centimeter or 1 per cubic millimeter. To resolve a single particle, the volume of the dielectric should be about 1/10 cu. mm. A parallel plate condenser with plates $\frac{1}{2}$ by $\frac{1}{3}$ mm. separated $\frac{1}{2}$ mm. will enclose 0.125 cu. mm., slightly more than 1/10 cu. mm. The capacity of such a condenser with air dielectric (K = 1.00059), will be

$$C = \frac{1.00059 \times 0.25}{3.6 \pi \times 0.5} = 0.0442 \ \text{micromicrofarads} \qquad (6)$$

The change of capacitance of this unit due to the presence of a cloud particle may be approximately determined by computing the change of stored energy in the system due to the presence of a dielectric

sphere.

If a dielectric sphere is placed in a uniform field whose electric intensity at large distances is $\bar{E}_o$, there will exist in the sphere a uniform field E_1 whose magnitude is given by the equation

$$E_1 = \frac{3e_o E_o}{2e_o + e_1}$$

where e_o is the specific inductive capacity of air

and e_1 is the specific inductive capacity of the dielectric sphere

(7)

According to Smythe (32), if the electric field intensity produced in the volume v of a uniform isotropic medium of specific inductive of capacity e_o is E_o, and if when v is occupied by a uniform isotropic body of specific inductive capacity e, it is E_1, then the change in stored energy in this volume is given by

$$\Delta W = \frac{1}{2} \int_V (e_1 - e_o)(E_o E_1) dv$$

(8)

Substituting equation (7) in equation (8) and performing the integration, the change in stored energy may be expressed

$$\Delta W = \frac{0.5 \ \pi (e_1 - e_o) \ e_o E_o^2 D^3}{2 \ (2e_o + e_1)}$$

where D = diameter of sphere

(9)

Since the capacitance of a system is specified by its stored energy, the following equation may be written

$$\frac{\Delta C}{C} = \frac{\Delta W}{W}$$

(10)

For a parallel plate condenser,

$$W = 0.5 \ e_o E_o^2 \ V_o$$

(11)

so that the incremental capacitance may be written

—

$$\Delta c = c \frac{\Delta W}{W} = \frac{c \pi (e_1 - e_0) D^3}{2(2e_0 + e_1)v_0} \qquad (12)$$

Substituting the numerical values for the problem under consideration,

$$\Delta C = 6.62 \times D^3 \times 10^{-10} \text{ micromicrofarads (mmf)} \qquad (13)$$

where D is in microns

Thus for a cloud particle whose diameter is 1 micron,

$$\Delta C = 6.6 \times 10^{-10} \text{ mmf} \qquad (14)$$

and for a cloud particle whose diameter is 100 microns,

$$\Delta C = 6.6 \times 10^{-6} \text{ mmf.} \qquad (15)$$

These changes would occur at an initial capacitance level of 4.42×10^{-2} mmf. The ratio of change would be 1 part in 10^8 for the smaller, 1 in 10^4 for the larger droplet.

A survey of commercial laboratory grade capacitance meters indicates that the lowest available inaccuracy is $0.1\% \pm 0.5$ mmf. at levels from 1 to 1000 mmf. The lower level of 1 mmf., is 10^2 above the initial value of our 1/10 cu. mm. condenser. It is 10 magnitudes removed from the accuracy required for detecting a single 1 micron droplet within the dielectric space of the condenser. However, it is only 7 magnitudes removed from the accuracy required for measuring the gross air/liquid water ratio in a much larger condenser of 1000 mmf. capacity.

There is described in the literature (22) a composite multi-range capacitance meter with a full scale range of 0.005 micromicro-farad and an indicating meter of probably 1/2 to 1% full scale inaccuracy. This instrument would measure one magnitude finer than the normal dry air value of our condenser, 0.0442 micromicrofarad, but 7 magnitudes

coarser than the change in value due to a single 1 micron drop, 6.6 x 10^{-10} mmf.

Thus it seems hardly possible that this method could be refined to detect single droplet in an airstream with presently available capacitance meters. Also the argument given above has neglected several important factors, all of which would introduce further errors in measurement, including

      1. leakage resistance across the condenser supports in the presence of water, both liquid and vapor,

      2. parallel capacitance of the supports and the attached electric circuits,

      3. temperature coefficients of capacitance and their corresponding errors due to variations in ambient temperature,

      4. change in dielectric constant with water vapor pressure.

The last of these sources of error may be separately considered as a possible means of measuring relative humidity. Consider the change in capacitance for dry and saturated air dielectrics in the condenser previously described. The constants for air and water vapor are 1.000,590

and 1.006,40, respectively, a difference of 0.005,81. At the freezing level, the maximum vapor pressure is 6.11 millibars. When the freezing level is at sea level, the maximum vapor pressure ratio is 6.11 mb./(1013.2-6.11) mb or 0.6%. The highest dielectric constant for saturated air at 0°C is then about 1.000,59 plus 0.6% of 0.005,81 or 1.000,625. To measure relative humidity to 1%, we must then measure

dielectric constant or capacity to 3 parts in 10 million at a level of 0.0442 micromicrofarads.

To compare the effect of saturated air to that of a single droplet, note that the capacity of the 0.125 cu. mm. condenser is

for dry air	0.044,200,000,00
for saturated air	0.044,201,550,00
for one 1 micron drop in saturated air	0.044,201,550,66

For relative humidity measurement, a much larger condenser than 0.125 cu. mm. might be considered. At 1 mf. initial dry air capacity, the least change measured for 1% R.H. change would be the same 3 parts in 10 million or 0.3 mmfd. The latter value by itself is readily measured, but as a change in the initial value, 1 mf., it is not measurable on any common capacitance meter.

The stability of most measuring circuits in the face of varying ambient temperatures, power sources and insulation surface leakages is not as good as 3 parts per million, either for long or short periods of time. However, a primary frequency standard such as made by the General Radio Company has a stability rating of a few parts in one hundred million over several months, and about two parts in one thousand million for short periods. One part in 10 million and 5 parts in 100 millions in 24 hours are typical short period ratings. Due to the nature of these piezoelectric frequency standards there are no random or cyclic variations of significantly larger amplitudes within the stated periods. It is conceivable, then, that an instrument using frequency changes controlled by capacity changes might be built for

relative humidity measurements in dry air or in cloud. Further consideration to this possibility will be given in a later report.

3. An investigation of the possibility of using the transmission of light as a means for measuring drop size distribution

It has been suggested that the drop size distribution in clouds could be obtained from light transmission measurements. This possibility is explored theoretically in the following section.

If an attenuation coefficient k is defined by the following equation

$$T = e^{-kZ} \tag{16}$$

where T is the ratio of incident to emergent intensity over a geometrical path length Z through the alternating medium, k must have a certain form depending in the process. If the process is pure scattering of light

$$k = \pi N_o \sum_{i=0}^{i=2} f_i r_i^2 \overline{K}_{s_i} \tag{17}$$

which is in accord with the Mie theory (25). N_o is the total number of spherical particles in a unit volume, f_i is the fraction of these drops whose radius is r_i and whose scattering cross section is K_{s_i}. The summation gives the contribution of all drops to the attentuation coefficient k.

As numerous investigators have pointed out (26), when the drop diameter d_i becomes relatively large compared to the wave length of the incident light, half of the scattered light may be assigned to the diffraction pattern and half to the rest of space. Now, as in this limit, $K_{s_i} \rightarrow 2$ for all wave lengths and the diffraction pattern accounts

for $\frac{1}{2} K_s$, the value of k for the entire diffraction pattern is (with $K_s = 1$)

$$k = \pi N_o \sum_{i=0}^{i=2} f_i r_i^2 \tag{18}$$

This limit is approached $\dfrac{d_i}{\lambda} \geqslant 5$

If all of the drops are uniform, $r_i = r$, a constant, and

$$\sum f_i r_i^2 = r^2 \sum f_i = r^2 \tag{19}$$

as

$$\sum f_i = 1. \tag{20}$$

Then

$$k = N_o \pi r^2 \tag{21}$$

and the attenuation coefficient is just equal to the total cross-sectional area intercepted per unit volume of the scattering medium.

If the entire diffraction pattern were not utilized in any measurement, but only a fraction thereof, equation (18) would not be satisfied and the apparent value of the attenuation coefficient, to be called k_a would always be less than k, except in the limit of the full diffraction pattern.

Let us define k_a in terms of the classical diffraction pattern.

For a given wave length of light λ and a given drop radius, r_i, the intensity of the diffraction patterns from an opaque circular disk is (17)

$$I_i = \pi^2 r_i^4 \left[\frac{1}{m_i} J_1 (2m_i) \right]^2 \tag{22}$$

where J_1 is the Bessel function of the 1st order and m_i is defined by

$$m_i = \frac{\pi r_i \sin \theta}{\lambda} \tag{23}$$

I_i thus gives the angular distribution of the diffraction pattern, θ being the angular spread of this same diffraction pattern.

When the diffraction pattern from a distribution of drops is considered, the intensity as a function of angle becomes

$$I = N_o \pi^2 \sum_{i=0}^{i=\infty} f_i r_i^4 \left[\frac{1}{m_i} J_1 (2m_i) \right] \tag{24}$$

where N_o and f_i have their usual significance.

The flux over a solid angle dw is given by

$$dF = I dw = 2\pi I \sin \theta \, d\theta \tag{25}$$

if the solid angle is a cone of revolution whose vertex angle is 2θ.

The total flux in the diffraction pattern over an angle θ is

$$F = 2\pi \int_0^\theta I \sin \theta \, d\theta = 2\pi \int_{\cos \theta}^1 I d(\cos \theta) \tag{26}$$

If we normalize this so that $F = 1$ when $\theta = \theta_{max}$, the angle that includes the entire diffraction pattern,

$$F (\theta) = \frac{\int_{\cos\theta}^1 I \, d(\cos \theta)}{\int_{\cos\theta_{max}}^1 I \, d(\cos \theta)} \tag{27}$$

where the denominator is the normalizing factor, after some cancellation of common factors has been carried out. Now as $0 \angle F \angle 1$, k_a can be defined as

$$k_a = \pi N_o F(\theta) \sum_{i=0}^{i=\infty} f_i r_i^2 \tag{28}$$

which reduces to equation (21) for $F (\theta) = 1$. The actual transmission is given with the aid of equation (16) as

$$T = e^{-\pi N_o Z \, F(\epsilon) \sum_{i} f_i r_i^2} \tag{29}$$

Experimentally, a transmission experiment could be set up, where the source and the receiver with a variable receiving area were at a fixed distance apart. If the distribution of drop sizes is known and T measured, $F(\Theta)$ is determined. One should note that $F(\Theta)$ is determined by the same distribution as $\sum_i f_i r_i^2$, and that $F(\Theta)$ as defined is an average intensity that will satisfy this distribution. As the relation might be used, however, the drop size distribution would be unknown, also $F(\Theta)$. The problem would then be to find a unique distribution that would satisfy equation (29). This might be done mathematically by assuming a distribution and computing $F(\Theta)$. If equation (29) were satisfied, the assumed distribution would be the correct one. This scheme would fail if it could be shown that one or several drop size distributions would give the same $F(\Theta)$ throughout the angular course of the diffraction pattern. Coincidence is assumed when there is no significant difference between the $F(\Theta)$'s.

Computation of $F(\Theta)$ Using Selected Drop Size Distributions

$F(\Theta)$ was computed for several drop size distributions, some of which were actual measured distributions using the glass slide technique.[4]

To compute $F(\Theta)$ from the data, equation (27) must be used after substitution of equation (24) into it to give

$$F(\Theta) = \frac{\int_{\cos\Theta}^{1} \sum_{i} f_i r_i^4 \left[\frac{1}{m_i} J_1(2m_i) \right]^2 d(\cos\Theta)}{\int_{\cos\Theta_{max}}^{1} \sum_{i} f_i r_i^4 \left[\frac{1}{m_i} J_1(2m_i) \right]^2 d(\cos\Theta)} \tag{30}$$

The quantity $f_i r_i^4 \left[\frac{1}{m_i} J_1 (2m_i) \right]^2$ is computed for all values, and summed

over all i's for a given angle θ. Tables of $\left[\frac{1}{m_i} J_1' (2m_i) \right]^2$ are available[17]

and f_i and r_i^4 come from the assumed distribution. The area under the

curve of $\sum_i f_i r_i^4 \left[\frac{1}{m_i} J_1 (2m_i) \right]^2$ versus $\cos \theta$ gives the numerator. The

summation is carried to the desired value of θ. This area divided by the

total area under the curve is $F(\theta)$. The integration was of necessity

carried out by graphical means. See Figure 1. $F(\theta)$ was also computed

for single drop sizes selected so as to produce an $F(\theta)/\theta$ curve nearly

identical to one computed from an actual distribution. Compare Figures

1 and 2. From this investigation it appears that there is little of

value in this method as a means for obtaining drop size distributions.

$F(\theta)$ was so lacking in uniqueness that this function as given by a

relatively broad drop size distribution could be matched to within a

few percent by an $F(\theta)$ as computed from a single drop size. This drop

size was very close to the area median drop size. The area median drop

size is defined by that drop size below which lies 50 per cent of the total

cross-sectional area presented by all of the drops. Symbolically

this is the value of r_i where

$$ \frac{\sum_{i=0}^{n} f_i r_i^2}{\sum_{i=0}^{\infty} f_i r_i^2} = 1/2 \tag{31} $$

As might be expected, those distributions with large numbers

of very small drops gave percentagewise large relative intensities $F(\theta)$

at large angles. Broad distributions were weighted in favor of the large

drops, so the diffraction pattern observable was relatively narrow.

The failure of this method probably lies to a large extent
in the slowness in which the angular spread of the diffraction pattern
varies with drop size. This allows a large spread of drop sizes to
contribute to the intensity pattern at a given angular distance.

4. A Further Study of the Relations between Light Transmission and Drop
   Size Distribution

In a previous section, it was shown that the light intensity $F(\theta)$
in a cone of variable angle θ from the axis of a diffraction pattern is
not measurably unique with drop size distribtuion. Very nearly equal
intensities may be received from a single sized drop cloud and from a
very wide size distribtuion. This is true for one wave length of light.
It is of interest to consider now the relation between transmission
and drop size distribution for a number of wave lengths over a wide
range.

Previous measurements (11) have shown no large variation in
transmission in the wave length range 1/3 to 3/4 micron. It is believed
that this range was not large enough to show in a measurable amount the
effect, for which some evidence of existence has been shown by unpublished
work of Houghton and by Sanderson (26).

The following procedure is proposed:

(1) measure transmission of monochromatic light of some 15 to
25 wave lengths in rapid succession.

(2) set up an equal number of simultaneous attenuation equations
for wave length and drop size, knowing

a. the scattering coefficient for each size range and

b. the absorption coefficient of water vapor in liquid-free

air for each wave length.

(3) solve the simultaneous equations for distributions of drop

size.

The equations to be used are as follows:

The transmission equation is

$$T = e^{-kz} \tag{32}$$

where T = transmissivity

z = path length

k = attenuation factor of medium

When the medium contains scattering spheres of different sizes,
equation (1) becomes

$$T = e^{-\pi z \sum nr^2 k_s} \tag{33}$$

where k_s = scattering coefficient

r = drop radius

n = number of drops of radius r

The scattering coefficient k_s is a function of the index of refraction
m and the absorptivity as well as of wave length λ and radius r. By
proper selection of wave length in the infrared, the absorptivity can
be reduced to such a value that the assumption of a transparent sphere
introduces only a small error. The value of k_s can be computed for any
value of m. It so happens that in the favorable wave length regions m
approaches 1.33. The relation of k_s to $\frac{2\pi r}{\lambda}$ for that index is well
known (15,30).

The accuracy of the method is improved by a better knowledge
of $k_s/2\pi r/\lambda$ for indices other than 1.33 and absorptivities other than
zero. However, the above method is a first approximation. Recomputations
are possible without repeated observations of transmission T whenever

superior values of k_s are available.

As part (2) of the procedure, equation (33) may be transposed to

$$\sum nr^2 k_s = \frac{\text{const}}{2} \ln \frac{1}{T} \tag{34}$$

The summation can be rewritten as

$$n_1 r_1^2 k_{s_1} + n_2 r_2^2 k_{s_2} + - - - - - - - - - - - - - - - - - - = \frac{\text{const}}{2} \ln \frac{1}{T} \tag{35}$$

At wave length λ_1

$$n_1 \text{const}_{1,1} + n_2 \text{const}_{1,2} + - - - - - - - - - - - - - - - - = \frac{\text{const}_{\lambda_1}}{2} \ln \frac{1}{T_1} \tag{36}$$

At wave length λ_{25}

$$n_1 \text{const}_{25,1} + n_2 \text{const}_{25,2} + - - - - - - - - - - - - - - - = \frac{\text{const}_{\lambda_{25}}}{2} \ln \frac{1}{T_1} \tag{37}$$

etc.

The transmissions would be measured 25 times, once at each wave length, at a rapid rate, such that no significant change in drop size distribution could occur during the measuring period.

The solution can be expressed as a summation of terms involving T of the form

$$n_1 = A_1 \ln \frac{1}{T_1} + A_2 \ln \frac{1}{T_2} + - - - - - - - - - - \tag{38}$$

$$n_2 = B_1 \ln \frac{1}{T_1} + B_2 \ln \frac{1}{T_2} + - - - - - - - - - - \tag{39}$$

where the coefficients A_n, B_n, etc. are known.

The mathematical evaluation would be relatively simple. The difficulty lies in measuring the transmission T in the 1 to 15 micron

range. Since water vapor absorption in certain bands gives additional attenuation over droplet scattering, it is necessary to select wave lengths so as to minimize such an effect. Little difficulty is expected in finding a sufficient number of well spaced nonabsorbing wave lengths and droplet size classes. Trial computations must be made for a limited number of values to determine the probable light energy levels and the required instrument sensitivities.

Before attempting drop size distribution measurement by this method, it is necessary to compute scattering coefficients for selected indices of refraction. The computation will require considerable research and experiment before a size distribution meter can be designed and constructed. Work on this preliminary step is now in progress.

D. Conclusions and Recommendations

Brief and preliminary studies have been made of several methods of detecting the presence, size and number of cloud droplets, by electric and light scattering means. Electrical effects which might form the basis of an instrument for measuring drop sizes and number appear to be smaller than the sensitivity that can be obtained by detectors now on the market or being developed.

The light scattering and attenuating methods so far considered are still unproved. Considerably more computations must be made before it can be said that the measuring methods we have in mind can or cannot be made to work.

It is recommended that our analysis of light attenuation in the infrared by scattering from small drops be carried further, to the

point where the magnitude of light attenuation and the probable require-
ments of instrument sensitivity are much better known. Meanwhile some
consideration should be given to the present state of commercial instrumen-
tation in order that orders may be placed for equipment as soon as there
is reasonable assurance of its usefulness.

D. P. Keily

III APPENDICES

APPENDIX A

Figures and Graphs

Figures 1 and 2 following are to be referred to section II,

C, 3.

APPENDIX B

Personnel and Administration

Staff members working on this project since its start on November 1, 1950 are

Delbar P. Keily, associate professor

John C. Johnson, assistant professor

Ralph G. Eldridge, research assistant
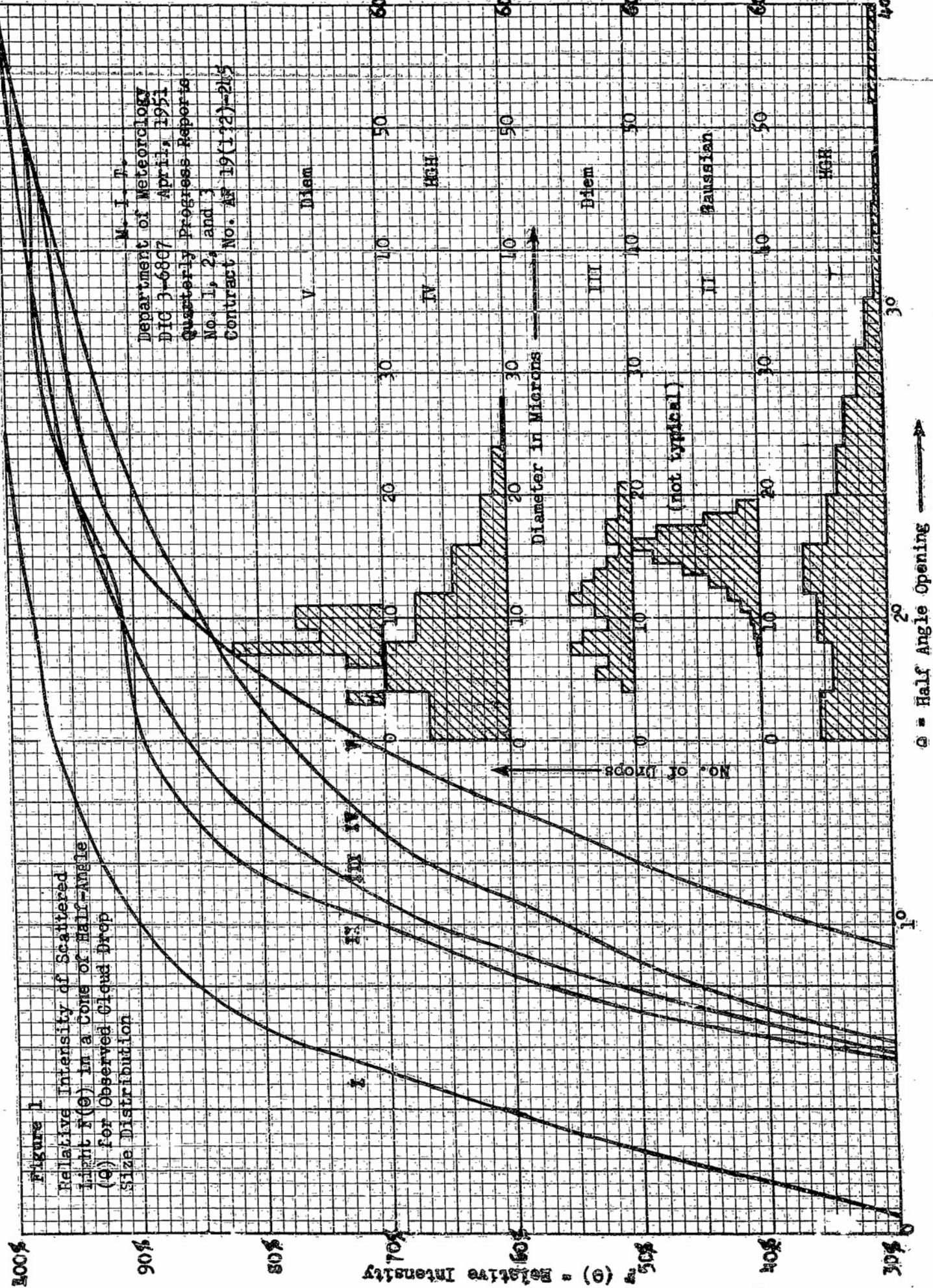
Richard M. Schotland, research assistant

Administrative action to date has been limited to employment of personnel and purchase of books and equipment.
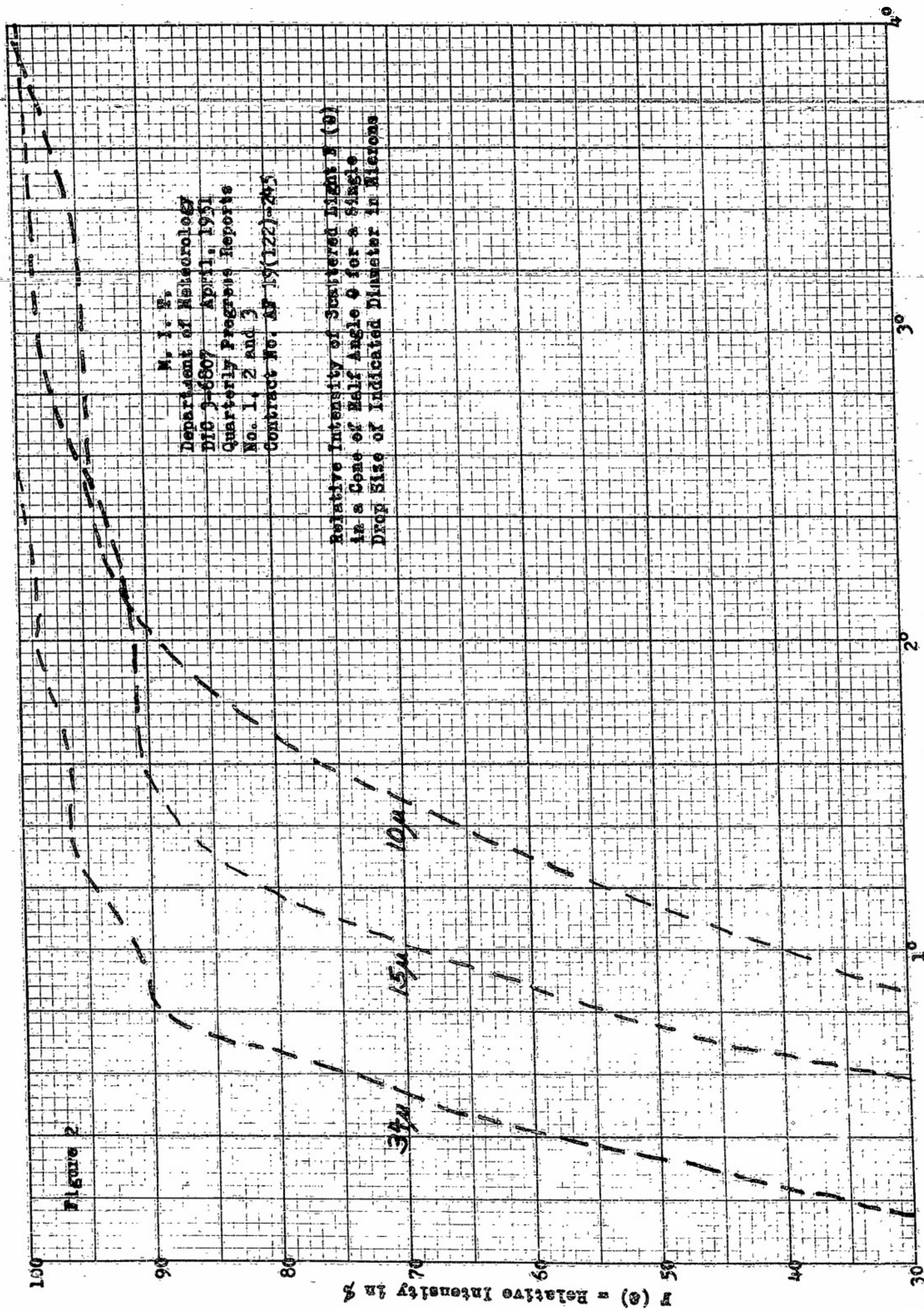
APPENDIX C

Travel and Correspondence

January 29, 1951 - R. M. Schotland and D. P. Keily attended the session on Physical Meteorology at the Meeting of the American Meteorological Society in New York.

January 30, 1951 - R. M. Schotland visited the laboratory of Professor V. K. La Mer at Columbia University. Electrical methods for measuring drop size and distribution were discussed. Information was obtained on sources of supply for technical equipment.

Figure 1

Relative Intensity of Scattered
Light $F(\Theta)$ In a Cone of Half-Angle
(Θ) For Observed Cloud Drop
Size Distribution

M. I. T.
Department of Meteorology
DIC 3-6807   April, 1951
Quarterly Progress Reports
No. 1, 2, and 3
Contract No. AF 19(122)-245

Θ = Half Angle Opening

Figure 2

M. I. T.
Department of Meteorology
DIC 2-6607   April 1, 1951
Quarterly Progress Reports
No. 1, 2 and 3
Contract No. AF-19(122)-245

Relative Intensity of Scattered Light $F(\theta)$
in a Cone of Half Angle θ for a Single
Drop Size of Indicated Diameter in Microns

Half Angle Opening in Degrees

$F(\theta)$ = Relative Intensity in %

APPENDIX B - BIBLIOGRAPHY

The following bibliography is intended to include the major references, both classic and recent, on drop counting and sizing, liquid water content, and allied subjects in cloud physics instrumentation. The recent references will lead in many instances to more complete lists on particular subjects.

The alphabetical author listing, A, is followed by a subject listing B, cross indexed to A.

A. Author listing.

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